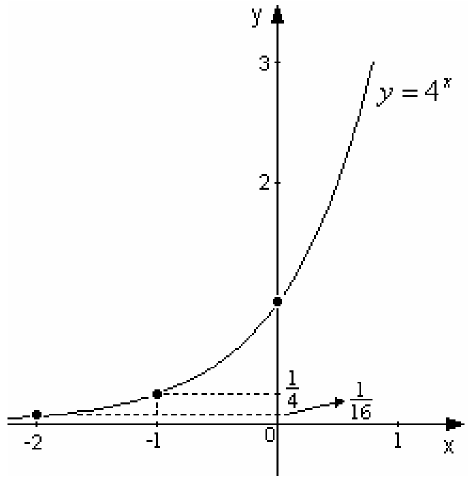
SOLUCIÓN A LOS EJERCICIOS DEL CAPÍTULO III

3.1. FUNCIÓN EXPONENCIAL

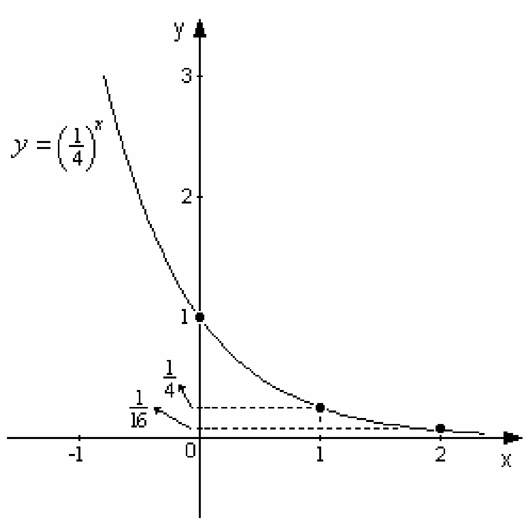
1) $y = 4^x$

x	4^x
2	16
1	4
0	1
-1	$\frac{1}{4}$
-2	$\frac{1}{16}$



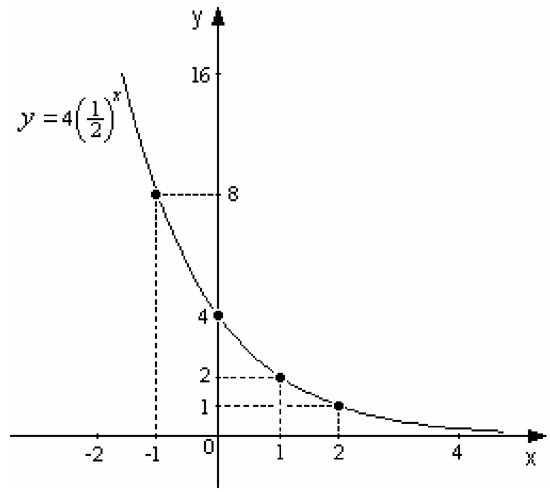
2) $y = \left(\frac{1}{4}\right)^x$

x	$\left(\frac{1}{4}\right)^x$
2	$\frac{1}{16}$
1	$\frac{1}{4}$
0	1
-1	4
-2	16



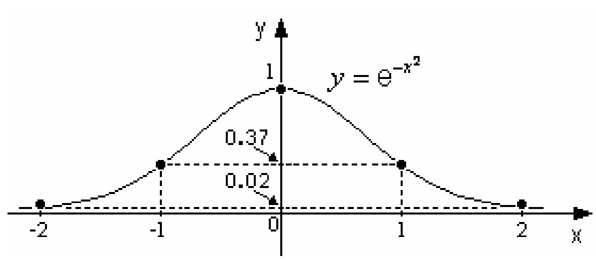
3) $y = 4\left(\frac{1}{2}\right)^{x}$

x	$4\left(\frac{1}{2}\right)^{x}$
2	1
1	2
0	4
-1	8
-2	16



4) $y = e^{-x^{2}}$

x	$e^{-x^{2}}$
2	0.02
1	0.37
0	1
-1	0.37
-2	0.02



5) $y = 79 + 6.4t - e^{3.25-t}$

Sustituyendo $y = 79 + 6.4t - e^{3.25-2} = 79 + 12.8 - e^{1.25} = 79 + 12.8 - 3.5$

$$y = 88.3[cm]$$

La estatura esperada del niño a los 2 años es de $88.3[cm]$

3.2. FUNCIÓN LOGARITMO

1) $y = \log x$

Cuando se representa el logaritmo de base 10, es costumbre escribir $y = \log x$ sin poner la base 10.

$y = \log x \quad \Leftrightarrow \quad x = 10^y$

x	y
0.001	-3
0.01	-2
0.10	-1
1	0
10	1
100	2
1000	3

Si

$$y = -3 \quad \Rightarrow \quad x = 10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$$

$$y = -2 \quad \Rightarrow \quad x = 10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$

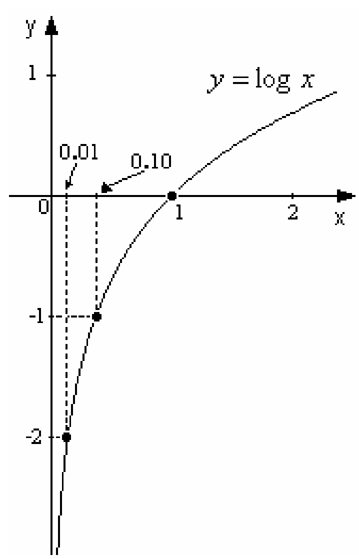
$$y = -1 \quad \Rightarrow \quad x = 10^{-1} = \frac{1}{10^1} = \frac{1}{10} = 0.1$$

$$y = 0 \quad \Rightarrow \quad x = 10^0 = 1$$

$$y = 1 \quad \Rightarrow \quad x = 10^1 = 10$$

$$y = 2 \quad \Rightarrow \quad x = 10^2 = 100$$

$$y = 3 \quad \Rightarrow \quad x = 10^3 = 1000$$



2) $y = \ln(2x)$

$y = \ln(2x) \quad \Leftrightarrow \quad 2x = e^y \; ; \; x = \frac{1}{2}e^y$

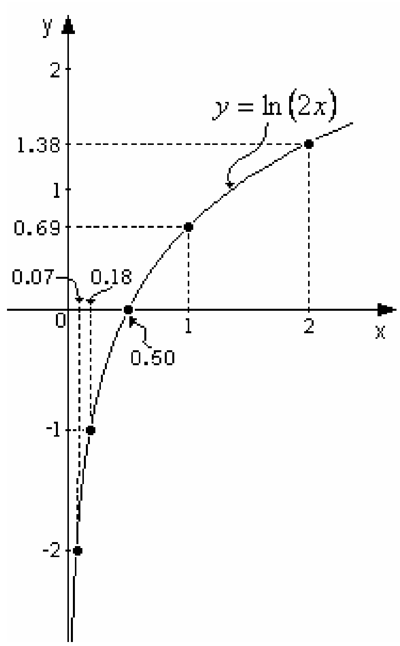
x	y
0.02	-3
0.07	-2
0.18	-1
0.50	0
1.00	0.69
2.00	1.38
10.04	3

Si

$$y = -3 \quad \Rightarrow \quad x = \frac{1}{2}e^{-3} = \frac{1}{2e^3} \cong 0.02$$

$$y = -2 \quad \Rightarrow \quad x = \frac{1}{2}e^{-2} = \frac{1}{2e^2} \cong 0.07$$

$$y = -1 \quad \Rightarrow \quad x = \frac{1}{2}e^{-1} = \frac{1}{2e^1} \cong 0.18$$



$y=0 \;\Rightarrow\; x=\frac{1}{2}e^{0}=\frac{1}{2}(1)=0.50$

$y=1 \;\Rightarrow\; x=\frac{1}{2}e^{1}\cong 1.36$

$y=2 \;\Rightarrow\; x=\frac{1}{2}e^{2}\cong 3.69$

$y=3 \;\Rightarrow\; x=\frac{1}{2}e^{3}\cong 10.04$

3) $y=\log_2 x^2$

Si $y=\log_2 x^2 \;\Leftrightarrow\; x^2=2^y\,;\, x=\pm\sqrt{2^y}$

$y=-3 \;\Rightarrow\; x=\pm\sqrt{2^{-3}}=\pm\sqrt{\frac{1}{2^3}}=\pm\sqrt{\frac{1}{8}}\cong\pm 0.35$

$y=-2 \;\Rightarrow\; x=\pm\sqrt{2^{-2}}=\pm\sqrt{\frac{1}{2^2}}=\pm\sqrt{\frac{1}{4}}\cong\pm 0.50$

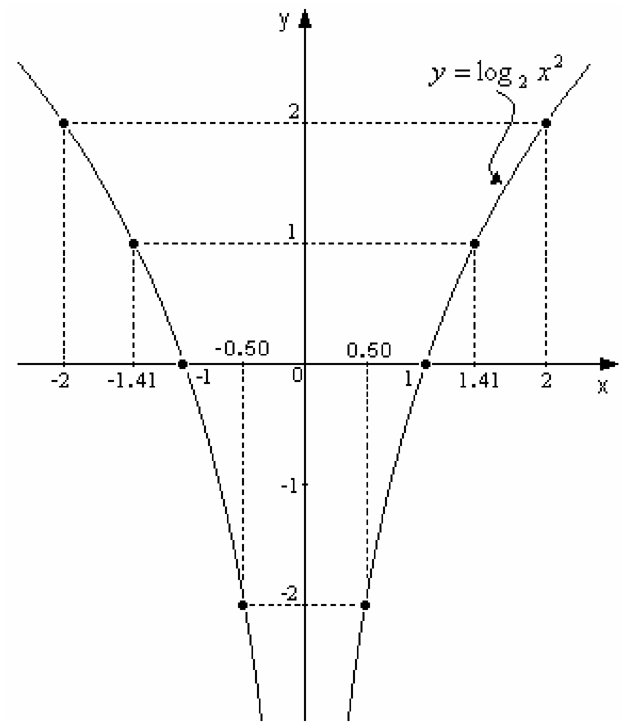
$y=0 \;\Rightarrow\; x=\pm\sqrt{2^{0}}=\pm\sqrt{1}=\pm 1.00$

$y=1 \;\Rightarrow\; x=\pm\sqrt{2^{1}}=\pm 1.41$

$y=2 \;\Rightarrow\; x=\pm\sqrt{2^{2}}=\pm\sqrt{4}=\pm 2$

$y=3 \;\Rightarrow\; x=\pm\sqrt{2^{3}}=\pm\sqrt{8}=\pm 2.83$

x	y
±0.35	-3
±0.50	-2
±0.71	-1
±1.00	0
±1.41	1
±2.00	2
±2.83	3



4) $y = \log_{\frac{1}{2}}\left(\frac{1}{2}x\right)$

$$y = \log_{\frac{1}{2}}\left(\frac{1}{2}x\right) \quad \Leftrightarrow \quad \left(\frac{1}{2}x\right) = \left(\frac{1}{2}\right)^y \; ; x = 2\left(\frac{1}{2}\right)^y = \frac{2}{2^y} = (2)\left(2^{-y}\right) = 2^{1-y}$$

x	y
16	-3
8	-2
4	-1
2	0
1	1
0.50	2
0.25	3

Si $y = -3 \quad \Rightarrow \quad x = 2^{1-(-3)} = 2^4 = 16$

$y = -2 \quad \Rightarrow \quad x = 2^{1-(-2)} = 2^3 = 8$

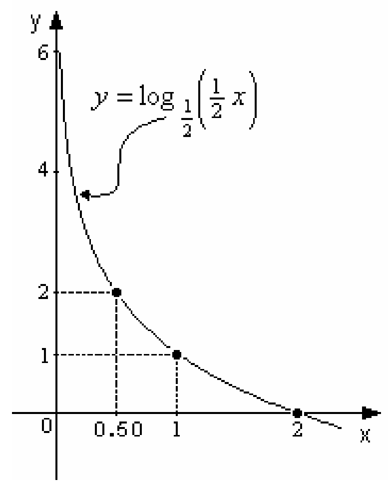
$y = -1 \quad \Rightarrow \quad x = 2^{1-(-1)} = 2^2 = 4$

$y = 0 \quad \Rightarrow \quad x = 2^{1-0} = 2^1 = 2$

$y = 1 \quad \Rightarrow \quad x = 2^{1-1} = 2^0 = 1$

$y = 2 \quad \Rightarrow \quad x = 2^{1-(2)} = 2^{-1} = \frac{1}{2} = 0.5$

$y = 3 \quad \Rightarrow \quad x = 2^{1-(3)} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4} = 0.25$



5) $y = \log_2(x+1)$

$y = \log_2(x+1) \iff (x+1) = 2^y \; ; x = 2^y - 1$

x	y
-0.88	-3
-0.75	-2
-0.50	-1
0	0
1	1
3	2
7	3

Si $y = -3 \Rightarrow x = 2^{-3} - 1 = \frac{1}{2^3} - 1 = \frac{1}{8} - \frac{8}{8} = -\frac{7}{8} = -0.88$

$y = -2 \Rightarrow x = 2^{-2} - 1 = \frac{1}{2^2} - 1 = \frac{1}{4} - \frac{4}{4} = -\frac{3}{4} = -0.75$

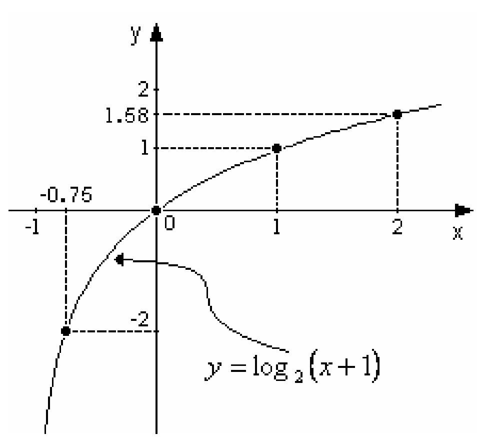
$y = -1 \Rightarrow x = 2^{-1} - 1 = \frac{1}{2^1} - 1 = \frac{1}{2} - \frac{2}{2} = -\frac{1}{2} = -0.50$

$y = 0 \Rightarrow x = 2^0 - 1 = 0$

$y = 1 \Rightarrow x = 2^1 - 1 = 1$

$y = 2 \Rightarrow x = 2^2 - 1 = 3$

$y = 3 \Rightarrow x = 2^3 - 1 = 7$



6)

$$M = C\left(1+\frac{i}{t}\right)^{tn}$$

Datos

C=$100 000
M=$300 000
t=1 año
i=20% anual

n=?

Cuando la variable por despejar es exponente, esto se logra aplicando logaritmos en ambos miembros de la igualdad.

$$\log M = \log C + tn\log\left(1+\frac{i}{t}\right)$$

$$n = \frac{\log M - \log C}{t\log\left(1+\frac{i}{t}\right)}$$

Sustituyendo valores se tiene:

$$n = \frac{\log 300\ 000 - \log 100\ 000}{1\log\left(1+\frac{0.20}{1}\right)} = 6.0256$$

por lo que convirtiendo los 0.0256 de años a días se tiene que:

$$\underline{n = 6\, años\ 9\, días}$$